كلية العلوم نموذج إجابة مادة: الفيزياء الاحصائية

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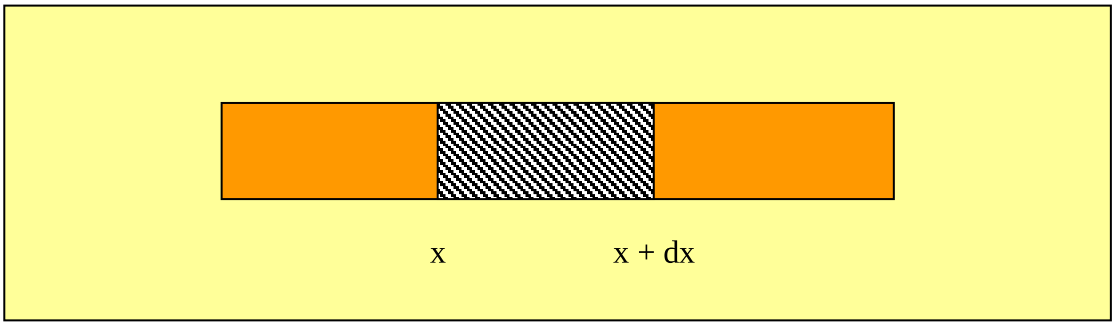
تاريخ الامتحان الإثنين 2013/06/09

1. Write a short note about the vibrational spectrum of crystals.

------------------- Solution ----------------------------

Let us examine the propagation of an elastic wave in a long bar. The wave equation in one dimension is



$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0 \qquad (1)$$

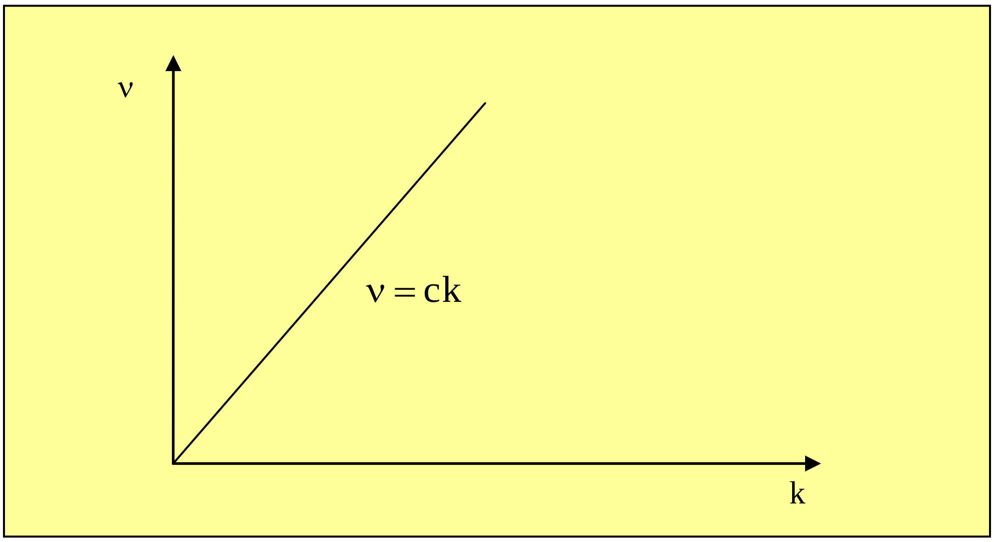
The solution of this equation is

$$\varphi = Ae^{i(kx-\nu t)} \qquad (2)$$

Substituting Eq. (2) in (1) leads to

$$\nu = ck \qquad (3)$$

The last equation is known as the dispersion relation which represents a straight line as in the figure

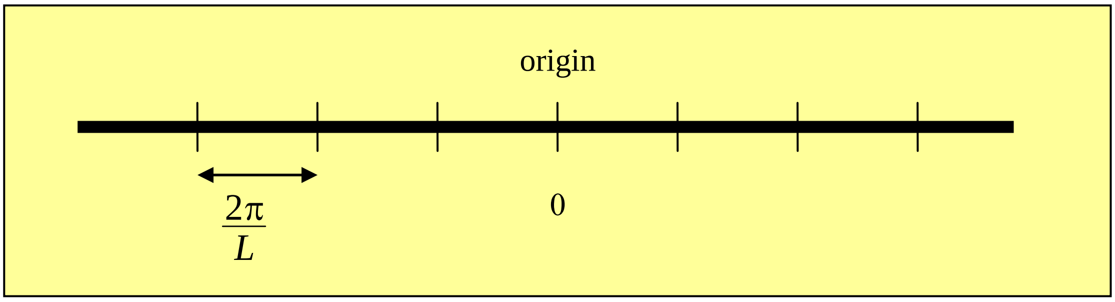


The boundary conditions require that

$$\varphi(0) = \varphi(L) \tag{4}$$

Substituting by Eq. (2) in (4) gives

$$k = n\frac{2\pi}{L}, \quad n = 0, \pm 1, \pm 2, \ldots \tag{5}$$



The density of states is

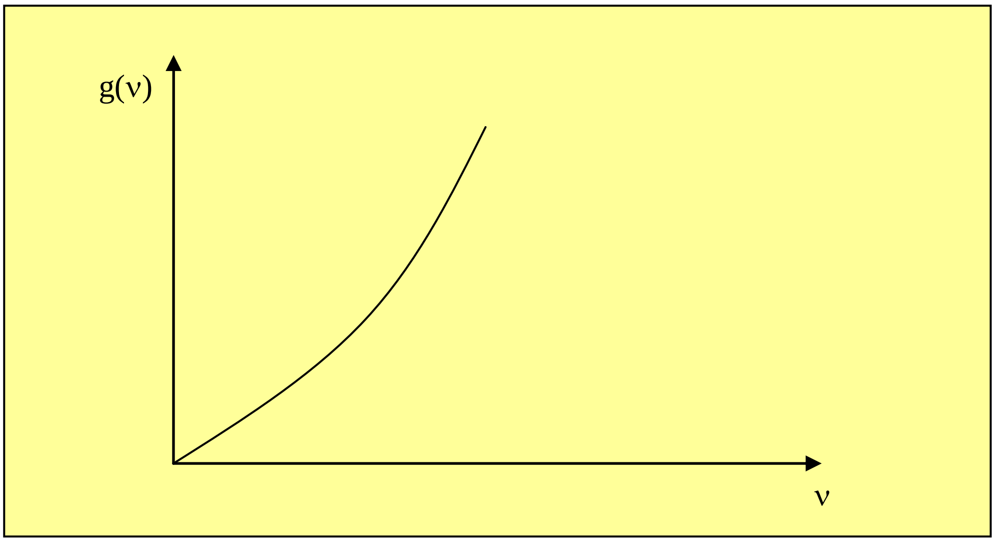
$$g(\nu)d\nu = \frac{L}{2\pi}dk \tag{6}$$

In one dimension

$$g(\nu) = \frac{L}{2\pi}\frac{1}{c}$$

In three dimension

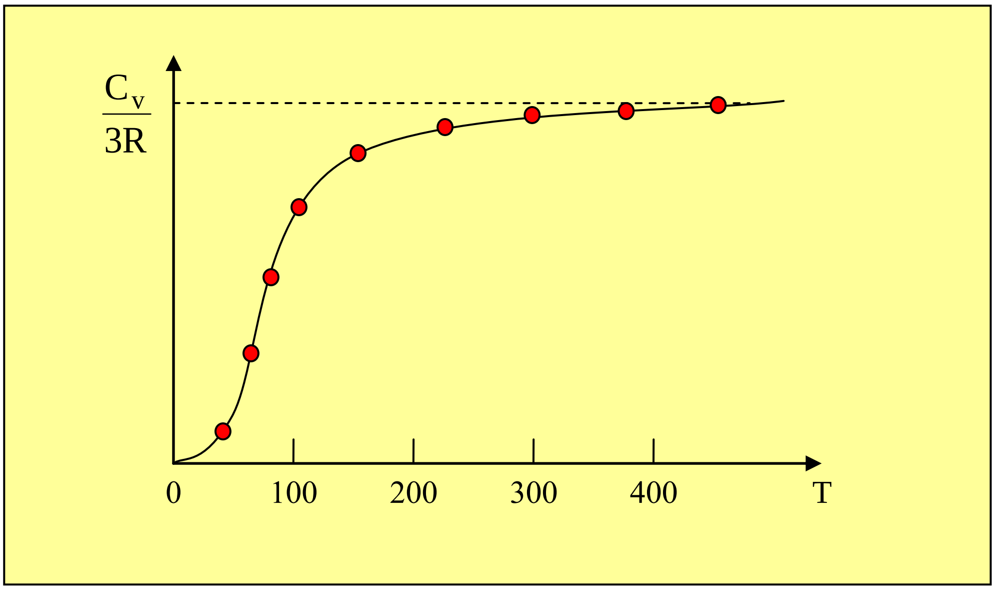
$$g(\nu) = \frac{3V}{2\pi^2}\frac{\nu^2}{c^3} \tag{7}$$


g(ν)
ν

2. Discus the classical theory interpretation for Duliong-Petit law of specific heat.

--------------------- Solution ----------------------------

The specific heat depends on the temperature as in the figure. At high temperature the value of C_v is close to 3R



In classical theory the average energy is

$$\bar{\varepsilon} = KT \quad (1)$$

And the energy per mole is

$$U = 3N_A KT = 3RT \quad (2)$$

So the specific heat at constant volume is

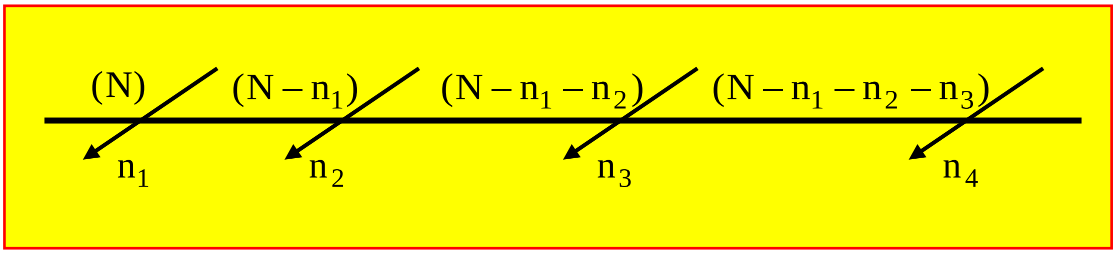
$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 3R \quad (3)$$

This is in agreement with experiment at high temperature, but it fails completely at low temperatures.

3. Prove the following relation for the occupation number n_i due to Bose - Einstein distribution $n_i = \dfrac{g_i}{e^{\alpha+\beta\varepsilon_i} - 1}$

-------------------- Solution ---------------------------

Let the number of allowed states associated with the energy ε_i be g_i. Let us first calculate the number of ways of putting n_1 particles of N particles in one box, then n_2 out of $N - n_1$ in second, and so on until we have exhausted all of the particles. The number of ways of choosing n_1 particles out of N particles is given by



$$W_1 = \frac{N!}{(N - n_1)!\, n_1!} \tag{1}$$

and the number of choosing n_2 out of $N - n_1$ is:

$$W_2 = \frac{(N - n_1)!}{(N - n_1 - n_2)!\, n_2!} \tag{2}$$

and the number of ways of achieving this arrangement is

$$W = W_1 \cdot W_2 \;\cdots$$

$$= \frac{N!}{(N - n_1)!\, n_1!} \cdot \frac{(N - n_1)!}{(N - n_1 - n_2)!\, n_2!} \cdots$$

$$= \frac{N!}{n_1!\, n_2! \quad \cdots \quad n_i!}$$

$$W = N! \prod_i \frac{g_i^{n_i}}{n_i} \tag{3}$$

$$\ln W = \ln N! + \sum_i (n \ln g_i - n \ln n_i!)$$
$$= N \ln N + \sum_i (n \ln g_i - n \ln n_i)$$

To obtain the most probable distribution, we maximize Eq. (3) with $dN = 0$:

$$\delta \ln W = \sum_i (\ln g_i - n \ln n_i - \frac{n_i}{n_i})\delta n_i = 0$$
$$\delta \ln W = \sum_i (\ln g_i - n \ln n_i - 1)\delta n_i = 0$$

but

$$\delta N = \sum_i \delta n_i = 0 \qquad (4)$$

$$\delta U = \sum_i \varepsilon_i \delta n_i = 0 \qquad (5)$$

multiply Eq. (4) by $\alpha + 1$ and Eq. (5) bt $-B$ and add the resulting equations to each other:

$$\sum_i (\ln g_i - n \ln n_i + \alpha - \beta\varepsilon_i)\delta n_i = 0 \qquad (6)$$

Since n_i is vary independent,

$$\ln g_i - n \ln n_i + \alpha - \beta\varepsilon_i = 0$$

or

$$\ln \frac{g_i}{n_i} + \alpha - \beta\varepsilon_i = 0 \qquad (7)$$

Solving Eq. (7) for n_i gives

$$n_i = \frac{N}{Z} g_i e^{-\beta\varepsilon_i}$$